Meshless Method Employing Magnetic Moment Method and Particle Method for Magnetic Fluid Motion Analysis

Kenta Mitsufuji[1], Katsuhiro Hirata[2], Fumikazu Miyasaka[3], Shuhei Matsuzawa[4]

[1]Dpt. of Adaptive Machine Systems, Osaka University, Osaka, Suita 565, Japan, kenta.mitsufuji@ams.eng.osaka-u.ac.jp
[2]Dpt. of Adaptive Machine Systems, Osaka University, Osaka, Suita 565, Japan, k-hirata@ams.eng.osaka-u.ac.jp
[3]Dpt. of Adaptive Machine Systems, Osaka University, Osaka, Suita 565, Japan, miyasaka@ams.eng.osaka-u.ac.jp
[4]Dpt. of Adaptive Machine Systems, Osaka University, Osaka, Suita 565, Japan, shuhei.matsuzawa@ams.eng.osaka-u.ac.jp

The Magneto Hydro Dynamics (MHD) phenomena show complex behavior. In this paper, a new meshless method employing the magnetic moment method and the particle method is proposed. The particle method is an analysis method that calculates fluid motion by dividing fluid into small particles. The magnetic moment method is an analysis method that calculates magnetic field by dividing ferromagnetic materials into small elements. In this study, fluid particles in the particle method are treated as ferromagnetic material elements in the magnetic moment method. Therefore, the proposed coupling method does not require calculation points in the air, remeshing in whole space. In this paper, the proposed method is applied to analyze the behavior of magnetic fluid under a magnetic field.

***Index Terms*— Numerical analysis, Meshless method, Particle method, Magnetic moment method, Magneto hydro dynamics.**

I. Introduction

MHD phenomena includes the complex behavior of fluid under a magnetic field. Especially, magnetic fluid [1]-[2] motions (e.g. spiking phenomenon) under a magnetic field involve a large deformation of fluid and an interaction between fluid and magnetic field. Therefore, it is difficult to simulate these phenomena. Recently, we proposed a coupling method employing the finite element method (FEM) and the particle method [3]. The coupled method made it possible to simulate MHD phenomena. However, FEM analysis requires calculation points in the analyzed space and remeshing process whenever the fluid moves. In Fig. 1, the mesh topology of divided fluid is shown. It is difficult to determine material type of elements between divided fluids in FEM analysis. The solution for this problem is replacing FEM with meshless magnetic field analysis method. The meshless method does not require meshes. This paper proposes a meshless method employing the particle method [4] and the magnetic moment method [5]. In this method, the magnetic moment method is employed to calculate the magnetic field and the particle method is employed to calculate the fluid motion under the magnetic field. The coupling algorithms and speeding-up method for the magnetic moment method are described. This method is applied to analyze the behavior a magnetic fluid.

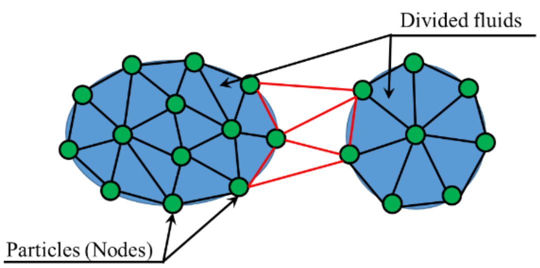

Fig. 1. The mesh topology of divided fluid.

II. Analysis Methods

The particle method is used for fluid motion analysis. In this method magnetic fluid is divided into small particles. The governing equations for incompressible fluid are as follows:

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\Delta\boldsymbol{u} + \boldsymbol{g} + \frac{\boldsymbol{f}}{\rho} \tag{1}$$

where D/Dt is Lagrangian derivative, $\boldsymbol{u}$ is velocity of fluid, P is pressure, ν is kinematic viscosity coefficient, $\boldsymbol{g}$ is acceleration of gravity, and $\boldsymbol{f}$ is force density. The equation (1) is solved under the condition of the mass continuity shown in (2).

$$\nabla \cdot \boldsymbol{u} = 0\,. \tag{2}$$

In this study, the force density $\boldsymbol{f}$ is magnetic force density calculated by the magnetic moment method.

The magnetic moment method is a magnetic field analysis method that solves a magnetization integration equation. The method does not need calculation points in the air space. The ferromagnetic material (fluid) is divided into small elements in magnetic moment method. In this method, the magnetic moment and magnetization are assumed to be constant in the ferromagnetic material elements. The interactions between ferromagnetic material elements are considered in the method as shown in Fig. 2. The interactions are expressed in a volume integral equation of magnetization. The volume integral equation for magnetization interaction is shown in (3).

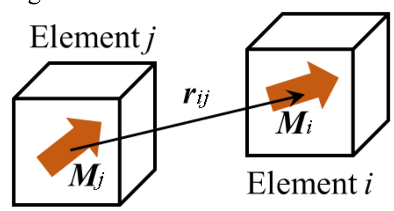

Fig. 2. The diagram of interaction of magnetization of ferromagnetic elements.

$$\boldsymbol{M}_i - \frac{\chi_i}{4\pi}\sum_{j=1}^{n}\nabla\left\{\int_V \boldsymbol{M}_j \cdot \nabla\left(\frac{1}{r_{ij}}\right)dV\right\} = \chi_i \boldsymbol{B}_{ci} \tag{3}$$

where $\boldsymbol{M}_i$ is the magnetization of ferromagnetic material element i, χ_i is magnetic susceptibility of ferromagnetic material element i, r_{ij} is the distance between ferromagnetic material element i and j, V is a volume of elements and $\boldsymbol{B}_{ci}$ is the magnetic density on gravity point of ferromagnetic element i in free space. Additionally, in Fig. 3, the magnetic field on arbitrary point P on the element of ferromagnetic material (magnetization: $\boldsymbol{M}$) is calculated by the following equation.

$$\boldsymbol{H} = -\frac{1}{4\pi\mu_0}\nabla\left\{\int_V \boldsymbol{M} \cdot \nabla\left(\frac{1}{r}\right)dV\right\}. \tag{4}$$

The μ_0 is magnetic permeability in free space, r is the distance between gravity point of the element and the calculation point P. In addition, contributions from multi ferromagnetic elements are superimposed on the calculation point P. After that, the magnetic force density at each particle in the particle method is calculated based on the magnetization vector and magnetic field vector obtained by the magnetic moment method.

III. The Coupling Method

As already mentioned, the particle method and magnetic moment method are coupled to calculate the behavior of a magnetic fluid. Magnetic field and magnetic forces are calculated by the magnetic moment method. Also, the particle motion is calculated by the particle method.

The fluid particles in the particle method are treated as magnetic material elements in the magnetic moment method in same position. Therefore, this method does not require model transformation between both methods. In this coupled method, each method are carried out seamlessly. Besides, magnetic moment method does not require calculation in the air space. Therefore, the method is robust to change in shape of fluid.

In magnetic moment method, a coefficient matrix of a linear system for solves (3) is dense matrix. Therefore, the speeding-up of magnetic moment method is required. In this study, the speeding up methods are clustering of interaction of magnetization and interpolation of magnetization on each particle using Weighted Least Square Method (WLSM). The details of this technique are explained in the full paper.

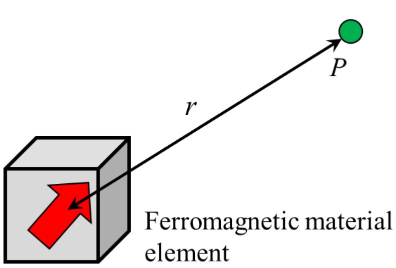

Fig. 3. Position relation of ferromagnetic element and arbitrary point P.

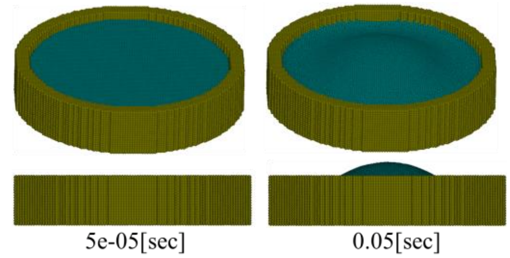

Fig. 4. Process of fluid height growth.

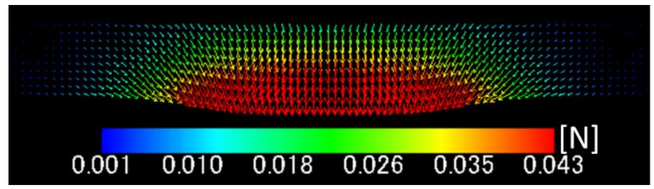

Fig. 5. Distribution of magnetic force vectors on cross section of fluid.

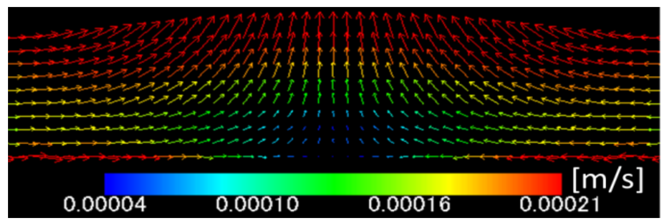

Fig. 6. Distribution of velocity vectors on cross section of fluid.

IV. Results and Discussion

Figs 4-6 show the analyzed results when this coupling method is applied to a magnetic fluid model. In this model, a columnar magnet (magnetization: 0.145T) is located under the magnetic fluid bottom. In Fig. 4, the magnetic fluid height increased as time passed. In Fig. 5, magnetic force vectors on particles verged to magnet. In Fig. 6, velocity vectors on particles verged to central upper part of a magnetic fluid. Also, on central lower part of a magnetic fluid, velocity on each particle are small. This behavior is occurred by convection of particles that are attracted to a magnet. From these results, analyzed results are qualitatively correct. The validity of the proposed coupled method for MHD analysis was confirmed. At the same time, the spiking phenomenon is not observed in this analysis result in Fig. 4. The condition of magnetic fluid model for occurring spiking phenomenon must be considered.

References

[1] A. Nethe, T. Scholz, H. –D. Stahlmann, and M. Filtz, "Ferrofluids in electric motors—A numerical process model," *IEEE Trans. on Magn.*, vol. 38, no. 2, pp. 1177-1180, Mar. 2002.

[2] H. S. Choi, Y. S. Kim, K. T. Kim, and I. H. Park, "Simulation of hydrostatical equilibrium of ferrofluid subject to magneto –static field," IEEE Trans. on Magn., vol44, no.6, pp.818-821, Jun. 2008.

[3] G. Yoshikawa, K. Hirata, F. Miyasaka and Y. Okaue "Numerical analysis of transitional behavior of ferrofluid employing MPS method and FEM," *IEEE Trans. on Magn.* Vol. 47, No. 5, pp. 1370-1373, May, 2011.

[4] S. Koshizuka and Y. Oka, "Moving-particle semi-implicit method for fragmentation of incompressible fluid," *Nucl. Sci. Eng. Soc.*, vol. 123, pp. 421-434, 1995.

[5] N. Takahashi, T. Nakata, N, Uchiyama, "Optimal design method of -3-D nonlinear magnetic circuit by using magnetization integral equation method," *IEEE Trans. on Magn*. Vol. 25, No. 5, pp 4144-4146, Sep, 1989.